SYNOPTIC: Stresses in Hollow Cylinders with Conical Ends Bonded to Shell and Shrunk,

A. J. Durelli, V. J. Parks, and K. Chandrashekhara, The Catholic University of America, Washington, D. C., *Journal of Spacecraft and Rockets*; Vol. 7, No. 8, pp. 964–967.

Solid Rocket Engines, Experimental Thermal Simulation, Thermal Stress, Structural Static Analysis

Theme

This paper deals with the three-dimensional photoelastic analysis of stresses and strains in two long, hollow circular cylinders of epoxy resin with conical ends inclined at 45° and 60°. The cylinders were bonded to steel casings to simulate case-bonded propellant grains. The stresses and strains due to the restrained shrinkage of the cylinders were determined at the interface. A natural, toroidal meniscus was formed at the open end of each cylinder. The strains at the meniscus surface were also studied.

Content

Figure 1 shows the dimensions of the cylinders, which were cast directly in the steel shells. Since the shells are rigid, the restrained shrinkage load can be considered as a uniform strain ϵ_t ($= \epsilon_2$) applied in the tangential direction on the bonded surface of the cylinders and a uniform displacement normal to the bonded surfaces. The uniform strain is assumed to be equal to α, the difference between the free shrinkages of the propellant and of the shell. The uniform displacement will be αr, where r is the radius of the bonded surface.

The epoxy formula used for the casting of the cylinders was the one suggested by Sampson. First, two resins (2 parts-by-weight Bakelite ERL-2274 and 2 parts-by-weight Bakelite ERL-2795) were heated to 180°F in separate containers. Then they were mixed, deaerated for 20 min at 27 in. Hg vacuum, and cooled to 90°F. Then hardener (1 part-by-weight Bakelite ZZL-0803) was poured slowly into the resin mixture and mixed thoroughly for about 10 min. The mixture was poured into the steel shells and was allowed to set for 24 hr. The complete system was heated in an oven to 170°F at 5°F/hr, then cooled to room temperature at 2°F/hr. A free disk and a ring shrunk on a plug were also cast at the same time for calibration.

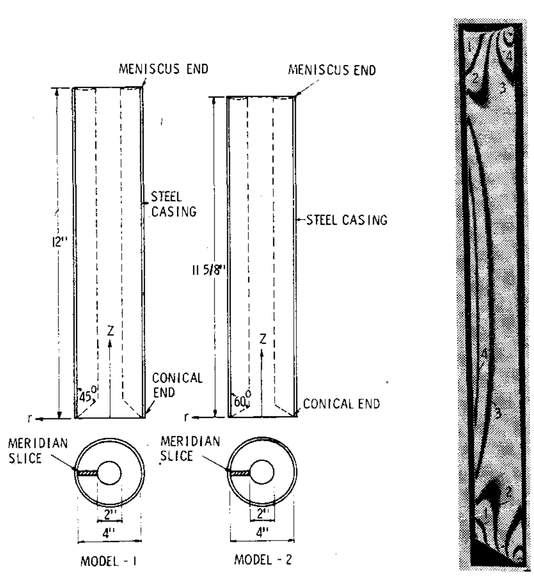

Fig. 1 Geometry and dimensions of hollow circular cylinders, and typical isochromatic pattern of the $\frac{1}{8}$-in.-thick meridian slice of model 2.

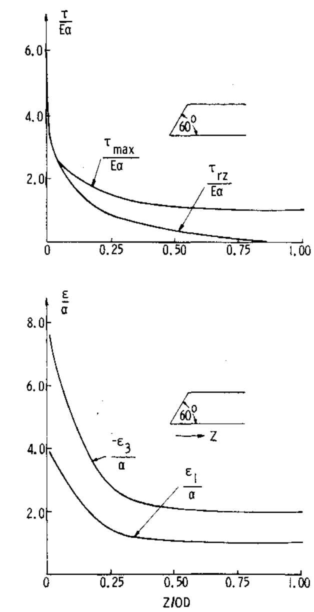

Fig. 2 Shear stresses and principal strain distribution along the interface in the cylinder with 60° cone.

A typical isochromatic pattern obtained from a meridian slice of the model with the 60° conical end is included in Fig. 1. Isoclinics near the corner were obtained for both models. Applying the stress-optic law, a tensor transformation, the boundary condition $\epsilon_t = \alpha$, and Hookes' law to the isochromatics and isoclinics, the results shown in Figs. 2 and 3 were obtained: normalized maximum shear stress $\tau_{\max}/E\alpha$, Cartesian shear stress $\tau_{tz}/E\alpha$ (where E is Young's modulus), and principal strains along the interface up to the conical end for each model. A similar analysis along the interface up to the meniscus end was also obtained for each model.

The highest stress $\sigma_{\max}$ in each model is $2(\tau_{\max})_{\max}$, and it occurs at the corner of the conical end on the bonded surface. For the cylinder with a 45° conical end, it is $\sigma_{\max} =$

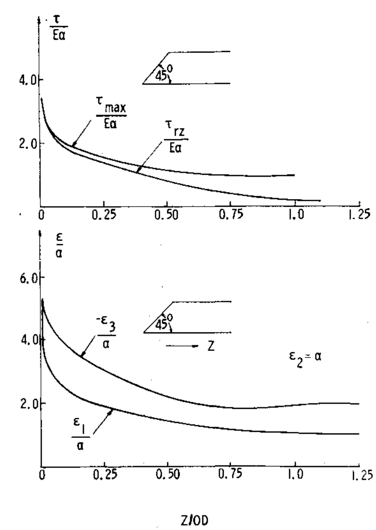

Fig. 3 Shear stresses and principal strain distribution along the interface in the cylinder with 45° cone.

$7.4\ E\alpha$, and for the cylinder with a 60° conical end it is estimated as $\sigma_{max} = 10.4\ E\alpha$. Although the meniscus profile is not the same in all the castings, the maximum stress at the meniscus end is often smaller than that at the conical corners and is estimated as $\sigma = 4.6\ E\alpha$.

As an aid to rocket grain designers these stress values are compared with a number of other plane-stress, plane-strain, and axisymmetric solutions in a table at the end of the paper. The other solutions are for the same load, restrained shrinkage, and for various geometries.

Stresses in Hollow Cylinders with Conical Ends Bonded to Shell and Shrunk

A. J. Durelli,* V. J. Parks,† and K. Chandrashekhara‡
The Catholic University of America, Washington, D. C.

This paper deals with the three-dimensional photoelastic analysis of stresses and strains in two long, hollow circular cylinders with conical ends of 45° and 60°. The cylinders were bonded to a steel casing on the outer surface. The stresses and strains due to restrained shrinkage have been determined at the interface. The maximum stress for both geometries was found to occur at the end of the bond, and to be about 40% higher in the 60° cylinder than in the 45° cylinder. A study was also made at the opposite end of the two cylinders, where a natural meniscus was formed. The results should be of interest in the design of solid-propellant rocket grains.

Introduction

PREVIOUS papers have reported results of an extensive study on stresses and strains in rectangular strips and plates, bonded on one face and shrunk[1] or subjected to biaxial restrained shrinkage,[2] and a study on the stresses in a square slab bonded on one face to a rigid plate and shrunk, which simulates a plane strain condition.[3,4] Cylinders with toroidal cavities also have been tested, and the results are reported elsewhere.[5] This paper is concerned with three-dimensional photoelasticity studies of the stresses and strains in two hollow circular cylinders having two different conical ends (at angles at 45° and 60°, respectively, with the bonded surface) and subjected to restrained shrinkage. As the shell is assumed to be rigid, the restrained shrinkage load can be considered as a uniform strain applied in the tangential direction on the bonded surface of the cylinder and a uniform displacement normal to the bonded surface. The uniform strain is specified as α and is equal to the difference between the free shrinkages of the propellant and of the shell. The uniform displacement will be αr when r is the radius of the bonded surface.

In the frozen stress technique that was used, thin slices were removed from the frozen stress model for analysis. The isochromatic patterns obtained give the maximum shear stress at any point in the slice and the maximum principal stress on free boundaries. The determination of the principal stresses and strains in the interior of the body and on the bonded surface requires further analysis. In this study the stress and strain distribution was obtained only along the bonded interface surface which is of particular interest as a region where failure is likely to occur. This type of failure at the interface is commonly referred to as bond failure, as opposed to the material failure which may occur in the interior of the body or on a free surface.

Each of the hollow circular cylinders used in this study (Fig. 1) was cast directly in a steel shell using the epoxy formula suggested by Sampson,[6] 2 parts-by-weight (pbw) of Bakelite resin ERL-2274, 2 pbw of Bakelite resin ERL-2795, and one pbw Bakelite hardener ZZL-0803. The two resins were heated to 180°F in separate containers, then mixed and deaerated for 20 min at 27 in. Hg vacuum. The mixture was then cooled to 90°F, and the hardener was added slowly and mixed thoroughly for ~10 min. The epoxy was poured into the steel molds and was allowed to set for 24 hr. Then the complete system was placed in the oven and was heated to 170°F at the rate of 5°F/hr. The epoxy and the mold were cooled to room temperature at 2°F/hr. A free disk and a ring shrunk on a plug were also cast at the same time for calibration.

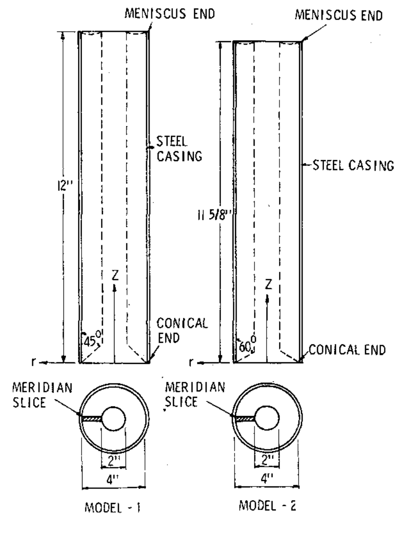

Fig. 1 Geometry and dimensions of hollow circular cylinders.

Received November 3, 1969; revision received May 12, 1970.

* Professor, Civil Engineering and Mechanics Department.

† Associate Professor, Civil Engineering and Mechanics Department.

‡ Postdoctoral Fellow, Civil Engineering and Mechanics Department; now Assistant Professor, Indian Institute of Science, Bangalore, India.